

Calculation of coils

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Calculation of coils

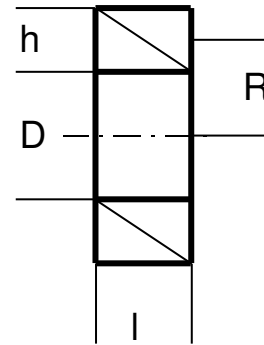
A : Formula

1. Short coils with multilayer winding and nonmagnetic core (neither iron nor ferrite)

$$L = \frac{21\mu_0\mu_r N^2 R}{4\pi} \left(\frac{R}{l+h} \right)^n$$

$$n = 0,75, \text{ if } \left(\frac{R}{l+h} \right) < 1$$

$$n = 0,5, \text{ if } 1 < \left(\frac{R}{l+h} \right) \leq 3$$



L	Vs/m ²	inductance
A	m ²	effective area, cross-sectional area of winding
d	m	diameter of the wire
D	m	diameter of the rod
R	m	mean radius of the coil
l	m	length of the coil
l*	m	length of the wire
h	m	height of the winding
N	-	number of the windings
μ ₀	Vs/Am	permeability of vacuum
μ _r	-	relative permeability

μ ₀	Vs/Am	4π 10 ⁻⁷	
μ _r	-	6 000 - 10 000	Permenorm [®]
μ _r	-	20 000	Ultraperm [®] 200
μ _r	-	5 000 - 50 000	Ultraperm [®] 10
μ _r	-	60 000 - 80 000	Vacoperm [®] 100
μ _r	-	20 000 - 40 000	Mumetall
μ _r	-	50 000 - 200 000	Ferrite
μ _r	-	1	air

2. Short coils with multilayer winding and nonmagnetic core (neither iron nor ferrite)

$$L = \mu_0 \mu_r N^2 \frac{6,2(D+h)^2}{3(D+h) + 9l + 10h}$$

3. Coil with multilayer winding and magnetic core

$$L = \mu_0 \mu_r N^2 \frac{A}{l}$$

4. Flux density

$$B = \frac{L \cdot I}{N \cdot A}$$

B	T	flux density 1T = 1Vs/m ²
L	Vs/A	inductance
I	A	electrical current
N	-	number of the windings
A	m ²	effective area, cross-sectional area of winding

5. Resistance of a coil

$$R = \rho \frac{l^*}{q}$$

R	Ω	resistance
l*	m	length of the wire
q	m ²	cross-sectional area of the wire
ρ	Ω mm ² /m	specific resistance
ρ _{co}	Ω mm ² /m	0,0178 for copper

6. Ohm's law

$$U = IR$$

U	V	voltage
I	A	current
R	Ω	resistance

7. Electrical power

$$P = UI = \frac{U^2}{R} = I^2 R$$

P	$W = V \cdot A$	electrical power
U	V	electrical voltage
I	A	electrical current

B : Calculation

B1 : coil dimensions proposed by Jonathan

d	0,4	mm	diameter of the copper wire
D	15	mm	diameter of the core
l	30	mm	length of the coil
N*	250	-	desired number of windings
B*	< 500	nT	desired flux density

Calculation of L corresponding to formula 1 :

- Number of windings per layer

$$\frac{l}{d} = \frac{30mm}{0,4mm} = 75$$

- Number of layers corresponding to desired value of windings

$$N = 75 \cdot 3 = 225 \Rightarrow 3 \text{ layers}$$

- Height of the winding

$$h = 3 \cdot d = 3 \cdot 0,4 = 1,2mm$$

- Mean radius of the coil

$$R = \frac{D+h}{2} = \frac{15+1,2}{2} = 8,1mm$$

- Test for calculation of the inductance

$$\left(\frac{R}{l+h} \right) = \frac{8,1}{30+1,2} = 0,2569 < 1 \quad \Rightarrow n = 0,75$$

$$(0,2569)^{0,75} = 0,2636$$

- Inductance of the coil

$$L = \frac{21\mu_0\mu_r N^2 R \left(\frac{R}{l+h} \right)^n}{4\pi} = \frac{21 \cdot 4\pi \cdot 10^{-7} \cdot 1 \cdot 225^2 \cdot 8,1}{4\pi \cdot 10^3} 0,3636 = 3,13 \cdot 10^{-4} \frac{Vs}{A}$$

Calculation of L corresponding to formula 2 :

$$L = \mu_0\mu_r N^2 \frac{6,2(D+h)^2}{3(D+h)+9l+10h} = \frac{4\pi \cdot 10^{-7} \cdot 1 \cdot 225^2 \cdot 6,2(15+1,2)^2}{3(15+1,2)+9 \cdot 30+10 \cdot 1,2} \cdot \frac{1}{10^3} = 3,13 \cdot 10^{-4} \frac{Vs}{A}$$

Calculation of flux density B

$$L = 3,13 \cdot 10^{-4} [Vs/A]$$

$$N = 225$$

$$A = \frac{\pi(D+h)^2}{4} = \frac{\pi(15+1,2)^2}{4} = 206 mm^2$$

$$B = \frac{L \cdot I}{N \cdot A} = \frac{3,13 \text{ Vs}}{10^4 \text{ A}} \frac{1}{2,06 \cdot 10^2 mm^2} \frac{10^6 mm^2}{1 m^2} \frac{I}{2,25 \cdot 10^2} = 6,7568 \cdot 10^{-3} \cdot I \left[\frac{Vs}{m^2} \right]$$

current I		flux density B	
1 A	1 A	6,7568 10^{-3} T	6,7568 mT
10^{-3} A	1 mA	6,7568 10^{-6} T	6,7568 μ T
10^{-6} A	1 μ A	6,7568 10^{-9} T	6,7568 nT
	66,6 μ A		450 nT

B2 : Alternative one - very small coil with diameter 2,2 mm

d	0,1	mm	diameter of the copper wire
D	2	mm	diameter of the core
l	1	mm	length of the coil
N	10	-	number of windings
h	0,1	mm	height of the winding

$$A = \frac{\pi(D+h)^2}{4} = \frac{\pi(2+0,1)^2}{4} = 3,46 mm^2$$

$$L = \mu_0\mu_r N^2 \frac{6,2(D+h)^2}{3(D+h)+9l+10h} = \frac{4\pi}{10^7} \frac{Vs}{Am} \cdot 1 \cdot 10^2 \frac{6,2(2+0,1)^2 mm^2}{[3(2+0,1)+9 \cdot 1+10 \cdot 0,1] mm} \frac{1 m}{10^3 mm} = 2,10 \cdot 10^{-7} \frac{Vs}{A}$$

$$B = \frac{L \cdot I}{N \cdot A} = \frac{2,10 Vs}{10^7 A} \frac{1}{10 \cdot 3,46 mm^2} \frac{10^6 mm^2}{m^2} I = 6,07 \cdot 10^{-3} I \left[\frac{Vs}{m^2} \right]$$

current I		flux density B	
1 A	1 A	$6,07 \cdot 10^{-3} \text{ T}$	6,07 mT
10^{-3} A	1 mA	$6,07 \cdot 10^{-6} \text{ T}$	$6,07 \mu \text{ T}$
10^{-6} A	$1 \mu \text{ A}$	$6,07 \cdot 10^{-9} \text{ T}$	6,07 nT
	74,1 $\mu \text{ A}$		450 nT

B3 : Alternative two – coil with diameter 20,2 mm

d	0,1	mm	diameter of the copper wire
D	20	mm	diameter of the core
l	1	mm	length of the coil
N	10	-	number of windings
h	0,1	mm	height of the winding

$$A = \frac{\pi(D+h)^2}{4} = \frac{\pi(20+0,1)^2}{4} = 317 \text{ mm}^2$$

$$L = \mu_0 \mu_r N^2 \frac{6,2(D+h)^2}{3(D+h) + 9l + 10h} = \frac{4\pi}{10^7} \frac{Vs}{\text{Am}} \cdot 1 \cdot 10^2 \frac{6,2(20+0,1)^2 \text{ mm}^2}{[3(20+0,1) + 9 \cdot 1 + 10 \cdot 0,1] \text{ mm}} \frac{1 \text{ m}}{10^3 \text{ mm}} = 4,447 \cdot 10^{-6} \frac{Vs}{\text{A}}$$

$$B = \frac{L \cdot I}{N \cdot A} = \frac{4,447 \text{ Vs}}{10^6} \frac{1}{\text{A}} \frac{10^6 \text{ mm}^2}{10 \cdot 317 \text{ mm}^2} \frac{1}{\text{m}^2} I = 1,40 \cdot 10^{-3} I \left[\frac{Vs}{\text{m}^2} \right]$$

current I		flux density B	
1 A	1 A	$1,40 \cdot 10^{-3} \text{ T}$	1,40 mT
10^{-3} A	1 mA	$1,40 \cdot 10^{-6} \text{ T}$	$1,40 \mu \text{ T}$
10^{-6} A	$1 \mu \text{ A}$	$1,40 \cdot 10^{-9} \text{ T}$	1,40 nT
	321,4 $\mu \text{ A}$		450 nT

B4 : Alternative three – coil with diameter 50,2 mm

d	0,1	mm	diameter of the copper wire
D	50	mm	diameter of the core
l	1	mm	length of the coil
N	10	-	number of windings
h	0,1	mm	height of the winding

$$A = \frac{\pi(D+h)^2}{4} = \frac{\pi(50+0,1)^2}{4} = 1970 \text{ mm}^2$$

$$L = \mu_0 \mu_r N^2 \frac{6,2(D+h)^2}{3(D+h) + 9l + 10h} = \frac{4\pi}{10^7} \frac{V_s}{\text{Am}} \cdot 1 \cdot 10^2 \frac{6,2(50+0,1)^2 \text{ mm}^2}{[3(50+0,1) + 9 \cdot 1 + 10 \cdot 0,1] \text{ mm}} \frac{1 \text{ m}}{10^3 \text{ mm}} = 1,219 \cdot 10^{-5} \frac{V_s}{\text{A}}$$

$$B = \frac{L \cdot I}{N \cdot A} = \frac{1,219 \text{ Vs}}{10^5 \text{ A}} \frac{1}{10 \cdot 1970 \text{ mm}^2} \frac{10^6 \text{ mm}^2}{\text{m}^2} I = 0,618 \cdot 10^{-3} I \left[\frac{V_s}{\text{m}^2} \right]$$

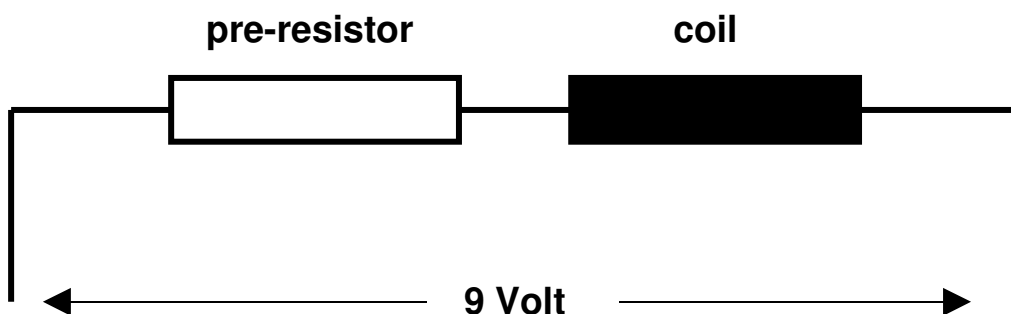
current I		flux density B	
1 A	1 A	0,618 10^{-3} T	0,618 mT
10^{-3} A	1 mA	0,618 10^{-6} T	0,618 μ T
10^{-6} A	1 μ A	0,618 10^{-9} T	0,618 nT
	728,1 μ A		450 nT

B5 : Resistances of the calculated coils

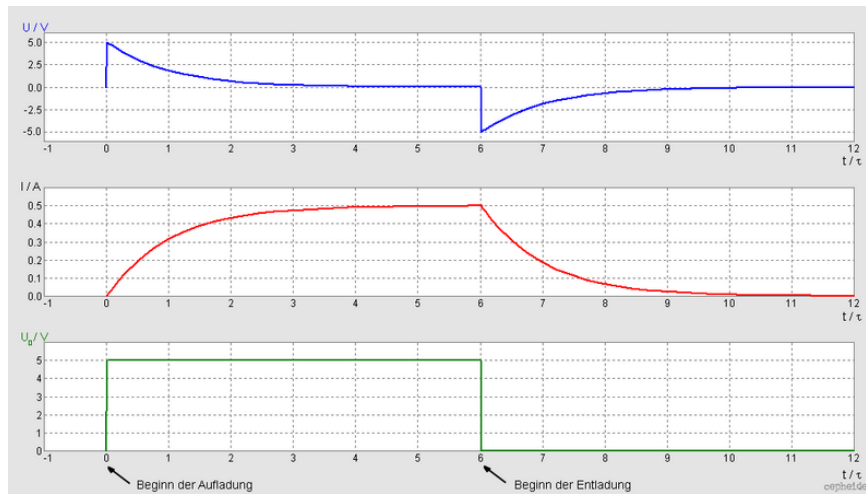
coil	$l^* = 2\pi R N$ [mm]	$q = \pi d^2/4$ [mm ²]	$R = \rho l^*/q$ [Ω]	$I = U/R$ [A] U = 9V	power [W]	comment
B1	11445,3	0,1256	1,622	5,55	49,95	non of the coils can be driven by these currents, power much too high
B2	62,8	0,00785	0,143	62,93	566,43	
B3	628	0,00785	1,43	6,29	56,64	
B4	1570	0,00785	3,56	2,53	22,75	

Necessary pre-resistor in line with the coil if U = 9V

coil	R_{coil} [Ω]	current μ A	$R_{\text{pre-resistor}}$ [k Ω]
B1	1,622	66,6	135
B2	0,143	74,1	121
B3	1,43	321,4	28
B4	3,56	728,1	12,4



B6 : Some remarks for running the circuit ¹



If a coil will be connected with a DC power supply, the electrical current changes according to an exponential function.

$$i(t) = \frac{U_0}{R} \left(1 - e^{-\frac{tR}{L}} \right)$$

L inductance of the coil

R resistance of the coil

U_0 DC-voltage supply

i electrical current

t time

That is why the induction voltage operates against the supply voltage. Hence this effect prevents a rapid rise of the electrical current.

It is not possible to switch off the electrical current according to a step. In the reality a voltage peak of reverse polarity develops when switching the current off. This voltage peak can take very high voltage values, much more than the supply voltage because of the differential

$$-\frac{di}{dt} \Rightarrow \infty$$

and can cause damage.

¹ [Spule \(Elektrotechnik\) – Wikipedia](https://de.wikipedia.org/wiki/Spule_(Elektrotechnik))

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